AAH-003-001617 Seat No. ________

B. Sc. (Sem. VI) (CBCS) Examination

March / April - 2016

BSMT-602(A) - Mathematics

(Mathematical Analysis-II & Group Theory-II)

Faculty Code : 003
Subject Code : 001617

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) All questions are compulsory
(2) All question of section – A carry equal marks and each question of section – B carry 25 marks.
(3) Write answer of each section in your main answer sheet.

SECTION – A

1 Write the correct answer of following MCQ's in your answer book : **20**

(1) The set E = (1,3) of metric space R is

(A) open (B) connected

(C) compact (D) Both (A) and (B)

(2) Which of the following is connected ?

(A) $R-\{3\}$ (B) $\{1, 2, 3, ..., 10\}$

(C) $(1,3)\cup(3,5)$ (D) None of these

(3) Two polynomial f and g in F are said to be associates if

(A) f and g are monic (B) f and g are non-zero

(C) f/g and g/f (D) None of these

(4) The set $E=\{x\in R/-3<x<0\}$ of metric space R is

(A) open (B) compact

(C) disconnected (D) closed

(5) A commutative division ring R is a field if

(A) $0 \in R$ (B) $1 \in R$

(C) $-a \in R, \forall a \in R$ (D) None of these

(6) If $\varphi : G \to G'$ is a homomorphism, where e and e' are identity of G and G' respectively then

(A) $\varphi(e) = 1$ (B) $\varphi(e) = e$

(C) $\varphi(e) = e'$ (D) None of these

(7) Which of the following is countable set ?

(A) N (B) Q

(C) R (D) Both (A) and (B)

(8) Every finite subset of any metric space is

(A) open (B) closed

(C) compact (D) Both (B) and (C)

(9) Laplace transform of e^{at} is

(A) $\frac{1}{a-s}$ (B) $\frac{1}{s-a}$

(C) $\frac{1}{s}$ (D) None of these

(10) If $\varphi : G \to G'$ is a homomorphism, K be the kernel then

(A) $G/K \cong G'$ (B) $G/K \cong \varphi(G)$

(C) $G/K \cong K$ (D) None of these

(11) Laplace transform of $t^{-1/2}$ is

(A) $\frac{\pi}{\sqrt{s}}$ (B) $\frac{\sqrt{\pi}}{s}$

(C) $\sqrt{\frac{\pi}{s}}$ (D) None of these

(12) $L(\sin at) =$ ______

(A) $\frac{a}{s^2 + a^2}$ (B) $\frac{s}{s^2 + a^2}$

(C) $\frac{s}{s^2 - a^2}$ (D) $\frac{a}{s^2 - a^2}$

(13) Laplace transform of t^n is, where $n = 0, 1, 2, 3, \ldots\ldots$

(A) $\frac{\gamma(n+1)}{s^{n+1}}$ (B) $\frac{n!}{s^{n+1}}$

(C) $\frac{\gamma(n)}{s^n}$ (D) Both (A) and (B)

(14) $L^{-1}\left\{\frac{1}{s}\right\} =$ ______

(A) t (B) t^2

(C) 1 (D) 0

(15) Laplace inverse of $\frac{1}{4s+5}$ is

(A) $e^{\frac{t}{4}}$ (B) $e^{-\frac{5t}{4}}$

(C) $\frac{1}{4}e^{-\frac{t}{4}}$ (D) $\frac{1}{4}e^{-\frac{5t}{4}}$

(16) Which of the following is not a field ?

(A) $(R, +, .)$ (B) $(C, +, .)$

(C) $(Z, +, .)$ (D) None of these

(17) Which of the following is an integral domain but not a field ?

(A) $(Z, +, .)$ (B) $(R, +, .)$

(C) $(Q, +, .)$ (D) None of these

(18) An integral domain R becomes a field if

(A) R is finite (B) R is singleton set

(C) R is infinite (D) None of these

(19) If $\varnothing:(G, X) \rightarrow (G', X)$ is a homomorphism where

$\varnothing = \{(2,0),(3,0),(4,1),(5,1)\}$ then $k_{\varnothing}$ is

(A) $\{1, 3\}$ (B) $\{1, 2\}$

(C) $\{4, 5\}$ (D) $\{2, 3\}$

(20) The polynomial $x^2 - 1$ is reducible over

(A) R (B) C

(C) Q (D) All of these

SECTION - B

2 (a) Attempt any **three** : **6**

(1) Define connected set and interval.

(2) Check whether the subset $E = \{1, 2, 3, ..., 11\}$ of R is compact or connected.

(3) If A and B are compact subsets of metric space R then show that $A \cap B$ is also compact.

(4) Find Laplace transform of $e^{-2t} \sin 5t$.

(5) Prove that $L\left[e^{at} \cosh bt\right] = \dfrac{s-a}{(s-a)^2 - b^2}$

(6) Prove that $L(\cos at) = \dfrac{s}{s^2 + a^2}$

(b) Attempt any **three** : **9**

(1) Prove that every open interval of metric space R is an open set.

(2) Show that the finite subset of a metric space is compact.

(3) Show that set $R-\{3\}$ is not connected.

(4) If $L\{f(t)\}=F(s)$ then prove that $L\left\{e^{at}f(t)=F(s-a)\right\}$.

(5) Find Laplace transform of $\cosh^3 2t$.

(6) Find inverse Laplace transform of $\frac{s}{\left(s^2-1\right)^2}$.

(c) Attempt any **two** : **10**

(1) If A and B are compact sets of metric space X then prove that $A\cup B$ and $A\cap B$ are also compact sets.

(2) State and prove theorem of nested intervals.

(3) Show that every compact subset of a metric space is closed set.

(4) If $L\{f(t)\}=\overline{f}(s)$ and $\frac{f(t)}{t}$ has Laplace transform then $L\left(\frac{f(t)}{t}\right)=\int_s^{\infty}\overline{f}(s)\,ds$.

(5) Find inverse Laplace transform of $\frac{1}{s\left(s^2+4\right)}$.

3 (a) Attempt any **three** : 6

(1) Define : Ring, Principle ideal ring.

(2) $\varnothing : (G, *) \rightarrow (G', \Delta)$ is Homomorphism. Then prove that $\varnothing\left(a^{-1}\right) = [\varnothing(a)]^{-1}$.

(3) Let l be an ideal of ring with unity R then prove that $l = R$ if $1 \in l$.

(4) Find the characteristics and zero divisor of ring z_6.

(5) $f(x) = (2, 3, 4, 2, 0, 0....)$ and $g(x) = (4, 2, 0, 0, 3, 0 ...)$ $\in z_5[x]$ then find $f(x) + g(x)$.

(6) Define : Principle ideal, Kernel of a homomorphism.

(b) Attempt any **three** : 9

(1) Show that $R = \left\{a + b\sqrt{2} / a, b \in z\right\}$ is a ring with respect to the usual addition and multiplication.

(2) $\varnothing : (G, *) \rightarrow (G' \Delta)$ is Homomorphism with kernel K_φ then prove that K_φ is a normal subgroup of G.

(3) Show that there does not exist proper ideal in Field.

(4) If l_1 and l_2 are any ideals of a ring R then prove that $l_1 \cup l_2$ is also an ideal.

(5) Give the example which is right ideal but not left ideal.

(6) In $R(x)$, $f(x) = 3x^4 - 5x^3 + 11x^2 + x - 1$ is divided by $g(x) = x^2 - 2x - 2$ then find quotient and remainder.

(c) Attempt any two : **10**

(1) Prove that a homomorphism $\varnothing:(G,*)\rightarrow(G'\Delta)$ is one-one iff $K_{\varphi}=\{e\}$.

(2) Prove that any integral domain is a field.

(3) State and prove Factor theorem and Remainder theorem.

(4) Prove that a commutative ring with unity is a field if it has no proper ideal.

(5) State and prove fundamental theorem of homomorphism.
